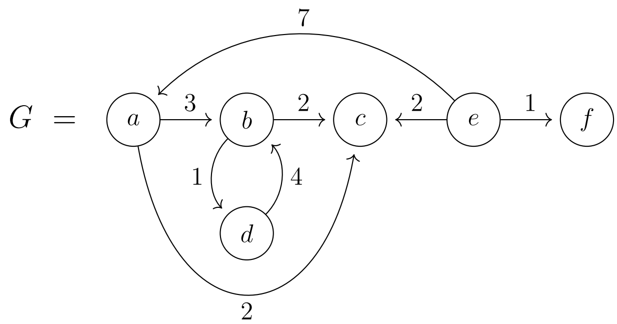
Assignment 10

Introduction to Linear Algebra

Material from Lectures 18 and 19
Due Thursday, March 23, 2023

18.1 (✠4.03, 4.04) Consider the following directed graph:



(a) Compute the adjacency A, incidence N, Laplacian L, and transition probability T matrices for G. Use the weighted definition for T.

(b) Row reduce N and give the resulting spanning tree of G.

(c) Using a computer make an educated guess as to what $\lim_{n\to\infty} T^n$ could be.

(d) Let $\hat{T}$ be the same as T, but with the (c, c) and (f, f) entries 1 (instead of 0) on the diagonal. Using a computer make an educated guess as to what $\lim_{n\to\infty} \hat{T}^n$ could be.

The last two questions address the *network flow*, where matrix multiplication represents movement along the edges, and the weights represent the probability of moving along a given edge (relative to all the weights outgoing from the tail node).

19.6 (✠3.01) Use the combinatorial definition of the determinant for this question. Recall from Example 19.15 that every term in the combinatorial definition uses exactly one entry from each row and one entry from each column of the matrix.

(a) Compute the determinant of

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

(b) The $n \times n$ identity matrix has $n^2 - n$ zeroes and exactly one nonzero term in the combinatorial definition. What is the smallest number of zeroes an $n \times n$ matrix can have so that the combinatorial definition has only one nonzero term?

19.3 (✠3.02) Let A be a 3×3 matrix. Suppose that $\det(A) = k$.

(a) Use the multilinearity property of the determinant to compute $\det(A + A)$.

(b) Use the multilinearity property of the determinant to compute $\det(-A)$.
Hint: Use the fact that $-A = A - 2A$.

(c) Explain how the result for part (b) would be different if A was a 2×2 matrix.