Introduction to Linear Algebra

Fall 2021 semester

Updated November 16, 2021

Instructor: Jānis Lazovskis `janis.lazovskis@rbs.lv`
Internal website: ORTUS
External website: `jlazovskis.com/teaching/linearalgebra`
Office hours (in person): Tuesdays 15:40 - 17:00 in Room 404
Office hours (on Zoom): Bookable at `calendly.com/jlazovskis`

About: This class will cover the basic topics of linear algebra, including vector spaces, the rank-nullity theorem, projections, determinants, diagonalization. Extensions to some topics will also be covered, including graphs, efficiently working with large matrices, optimization. Familiarity with matrices and vectors is assumed. A full list of topics is given on the next page.

This class is based on several courses, listed below. It is mostly the first course, with select material from the second course. Everything is arranged to fulfill the requirements of the third course.

- 18.06 Linear Algebra (MIT)
- 18.065 Matrix Methods in Data Analysis, Signal Processing, and Machine Learning (MIT)
- MTH 309 Introductory Linear Algebra (SUNY Buffalo)

Two extra lectures are reserved to account for difficult topics and / or topics relevant to the students. Some earlier lectures may be split up into two parts, so the schedule may shift slightly.

All the assigned work will follow a "scaffold" approach: In-class exercises will be used to create exercises in the homework, which in turn will be used to create questions for the midterm and final. The idea is that you should not be surprised by any questions on exams, and that you develop not only broad, but also deep knowledge.

Textbook: There is no required texbook for this course, as all the material will be given in the lecture notes. There are some suggested textbooks, from which definitions and exercises may be taken:

[1] (MIT) *Introduction to Linear Algebra*, Strang (2016)

[2] (MIT) *Linear Algebra and Learning from Data*, Strang (2019)

[3] (SUNY Buffalo) *Linear Algebra and Its Applications*, Lay, Lay, McDonald (2016)

[4] *Linear Algebra Done Right*, Sheldon Axler (2015). Very abstract and theoretical

[5] *Linear Algebra Done Wrong*, Sergei Treil (2017). Very concrete and constructive

Homework: Homework assignments will be given approximately once every two weeks. They are due at the beginning of the lecture. If you cannot complete the homework on time, please inform me and I will give you an extension. Some assigned work will be coding, in which case Python $\geqslant 3$ is the preferred language, unless otherwise noted. There will be no coding questions on the midterm or final.

Grading: Your final grade will be 50% homework, 15% midterm, 25% final, and 10% participation. Participation means attending the lectures, meaningful classroom interaction, and submitting the quizzes every lecture. The final exam will be take home, and you will have two weeks to complete it.

Academic integrity: You are encouraged to work together to complete assignments, but please write up your own solutions. The work you submit must be your own and should reflect your own understanding of the problem. Part of the RBS official stance on academic integrity is indcluded below.

RTU Riga Business School values intellectual integrity and the highest standards of academic conduct. To be prepared to meet societal needs as leaders and role models, students must be educated in an ethical learning

environment that promotes a high standard of honor in scholastic work. Academic dishonesty undermines institutional integrity and threatens the academic fame of RBS. Dishonesty is not an acceptable avenue to success. It diminishes the quality of RBS education, which is valued because of RBS high academic standards.

Fostering an appreciation for academic standards and values is a shared responsibility among students, faculty, and staff. RBS prohibits dishonesty in connection with any RBS activity. [..] A commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated.

Class schedule: Strang [1] will be used as the main source, but it is not available to RBS. The relevant sections in Lay [3] are also given. References to sections that are underlined refer to another Strang textbook [2].

Part	Date	Topic	Strang	Lay	Activities
I. Vector spaces	Aug 31	1: Vectors and matrices	1.1 - 1.3	1.1 - 1.5	HW 1 released
	Sep 3	2: Elimination and inverses	2.1 - 2.7	2.1 - 2.5	
	Sep 7	3: The column space and nullspace	3.1, 3.2	4.1, 4.2	
	Sep 10	4: Completely solving $A\mathbf{x} = \mathbf{b}$	3.3	-	HW 1 due
	Sep 14	5: Independence, basis, dimension	3.4	4.3, 4.4, 4.7	HW 2 released
	Sep 17	6: The rank-nullity theorem	3.5	4.6	
	Sep 21	7: Orthogonal spaces	4.1	6.1 - 6.3	
II. Operations	Sep 24	8: Projections and least squares	4.2, 4.3	6.5, 6.8	HW 2 due
	Sep 28	9: The Gram–Schmidt process	4.4	6.4	HW 3 released
	Oct 1	10: Inner products and distances	IV.10	6.7	
	Oct 5	11: Determinants, part 1	5.1, 5.2	3.1	
	Oct 8	12: Determinants, part 2	5.2, 5.3	3.2, 3.3	HW 3 due
	Oct 12	13: Eigenvalues and eigenvectors	6.1	5.1, 5.2, 5.4	
	Oct 15				**Midterm**
III. Deconstructions	Oct 19	14: Diagonalization	6.2	5.3, 5.6	
	Oct 22	15: Special matrices	6.4, 6.5	7.1, 7.2	HW 4 released
	Oct 29	16: Singular value decomposition	7.1, 7.2	7.4	
	Nov 2	17: Principal component analysis	7.3, 7.4	7.5	
	Nov 5	18: Linear transformations	8.1, 8.2	1.8, 1.9	HW 4 due
	Nov 9	19: Jordan form	8.3	-	HW 5 released
	Nov 12	20: Complex numbers and matrices	9.1, 9.2	5.5	
IV. Extensions	Nov 16	21: Fourier topics	8.3, 9.3	6.8	
	Nov 23	22: Graphs and networks	10.1	-	HW 5 due
	Nov 26	23: Markov matrices, spectral analysis, clustering	10.3, IV.6, IV.7	10.4 - 10.6	HW 6 released
	Nov 30	25: Large matrices and randomization	II.4	-	
	Dec 3	26: Optimization	10.4, VII.1	9.2, 9.3	HW 6 due, Final exam released
	Dec 17				**Final exam due**