Midterm topics

Introduction to Linear Algebra

Material from Lectures 1 - 12

Fall 2021

You should have the following skills from each lecture.

1. **Vectors and matrices**
   - Add, subtract, multiply vectors and matrices
   - Apply properties of triangular and block matrices, operations on them
   - Compute length of vectors and angle betwee vectors
2. **Gaussian elimination and inverses**
   - Apply Gaussian and Gauss–Jordan elimination
   - Decompose a matrix A by the LU-decomposition
3. **The column space and the nullspace**
   - Determine whether or not something is a vector space
   - Compute the reduced row echelon form of a matrix, find its inverse this way
4. **Completely solving $A\mathbf{x} = \mathbf{b}$**
   - Find the pivot columns, free columns, rank of a matrix
   - Construct the complete solution to a matrix equation
5. **Independence, basis, dimension**
   - Construct, identify a basis of vectors for a vector space
   - Construct a change of basis matrix
   - Combine vector spaces to make new ones (intersection, sums)
6. **The rank-nullity theorem**
   - Find the four fundamental subspaces of $A \in \mathcal{M}_{m\times n}$ as spans of linearly independent vectors
7. **Orthogonal spaces**
   - Determine if vectors are independent, orthogonal, orthonormal
   - Apply the properties of orthogonal complements of vector spaces
8. **Projections and least squares**
   - Compute the projection of vectors onto subspaces and their complements
   - Compute the least squares approximation for degree-k polynomials

9. **The Gram–Schmidt process**

    - Apply the Gram–Schmidt process to a set of linearly independent vectors
    - Decompose a matrix A by the QR-decomposition

10. **Inner products and distances**

    - Compute the inner product in an arbitrary inner product space
    - Construct a distance matrix from given vectors

11. **Determinants, part 1**

    - Apply the recursive and pivot definitions of the determinant
    - Use all properties of the determinant

12. **Determinants, part 2**

    - Compute the matrix of cofactors
    - Apply the permutations definition of the determinant