Discrete Structures

Worksheet 26

Spring 2022
April 8

Recall the following terms in probability:

- The phrase "**random variable**" is used to describe events whose outcome can not be directly predicted. It is used like:

  "If the number rolled on a fair die is given by X, then the probability that $X = 5$ is $\frac{1}{6}$."

- The **expected value**, or **mean**, of a random variable X is the sum of all the outcomes, where each outcome is multiplied by its probability. It is written $E[X]$ or μ.
- The **variance** of a random variable is the expected value of the random variable $(X-\mu)^2$. The variance $V(X)$ is the square of the **standard deviation** σ.

Calculate the expected value and variance of the following random variables. Draw the graph of the probability function of X. To draw the graph, you may use a calculator.

1. There are N pieces of paper in a bag, labeled with the numbers $1, \ldots, N$. Of these, $n \leqslant N$ are drawn without replacement. Let X be the largest number selected.

2. There is a fleet of 200 trucks, of which 12 have defective brakes. In a safety check, 10 are picked at random for inspection. Let X be the number of trucks with defective brakes that are chosen for inspection.

3. Someone randomly throws a dart at a circular piece of paper with radius 8cm. Let X be the distance from the center of the paper.

4. A dartboard has concentric circles, with radii 1cm, 2cm, ..., 8cm. Landing a dart between circles of radii $n-1$ and n cm gives n points. Someone randomly throws a dart at the dartboard. Let X be the number of points received.

5. The setup is the same as in Question 4. This time, the person throwing the dart has a probability of $\frac{2\sqrt{8-r}}{15\pi}$ of being a radius $r$cm away from the center. Let X be the number of points received.