

1. AA batteries are shipped to stores for sale in boxes containing 500 batteries. Before shipment each box must be tested for quality control purposes. Inspector Bob has three methods for testing:
  - Method A: Select 20 batteries for testing and reject the box for shipment if more than 1 defective battery is found.
  - Method B: Select batteries for testing until 20 non-defective batteries are found. If fewer than 2 defective batteries are found then the box is shipped.
  - Method C: Select 40 batteries for testing and reject the box for shipment if more than 2 defective batteries are found.

Suppose there are actually 10 defectives in the box.

- (a) Find an expression for the probability that the box is rejected using Method A.
  - (b) Find an expression for the probability that the box is rejected using Method B.
  - (c) Find an expression for the probability that the box is rejected using Method C.
  
2. A necklace consists of 5 beads on a string. The beads for making the necklace are drawn at random from a box containing a very large number of beads. Two-thirds of the beads are pink and one-third of the beads are blue. Find the mean and variance of the number of unlike pairs of adjacent beads in the necklace.

*Hint: Let  $X_i = 1$  if bead  $i$  and  $i + 1$  have different colors, and 0 if they have the same colors.*
  
3. A jar contains a large number of jelly beans. Suppose one third of the jelly beans are red, one half are green, and the rest are blue. You plan to randomly select 20 beans from the jar. Let  $R$ ,  $G$ , and  $B$  be the numbers of red, green, and blue beans you select.
  - (a) What is the probability that  $R = 7$  and  $G = 10$ ?
  - (b) You plan to buy the red beans, at a cost of 6 cents each, and the green beans, at a cost of 4 cents each (you will not buy the blue beans). So your total cost will be  $C = 6R + 4G$  cents. Compute the expected value of  $C$ .
  
4. If you take a bus to work in the morning there is a 20% chance you'll arrive late. When you go by bicycle there is a 10% chance you'll be late. 70% of the time you go by bike, and 30% by bus.
  - (a) What is the probability that you will be late?
  - (b) Given that you arrive late, what is the probability you took the bus?
  - (c) If you have two bicycles  $A$  and  $B$ , which you use with equal probability, what is the probability that you took bicycle  $B$ , given that you are on time?

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5. Complete the following tasks for next lab (Friday). They will be presented at the beginning of the lab.

(a) A manufacturer produces bolts that are specified to be between 1.19 and 1.21 cm in diameter. If his production process results in a bolt's diameter being normally distributed with mean 1.20 cm and standard deviation 0.005, what percentage of bolts will not meet specifications?

(b) An experiment has three possible outcomes  $A$ ,  $B$  and  $C$  with respective probabilities  $p$ ,  $q$  and  $r$ , where  $p + q + r = 1$ . The experiment is repeated until either outcome  $A$  or outcome  $B$  occurs. Show that  $A$  occurs before  $B$  with probability  $p/(p + q)$ .

(c) A researcher wishes to estimate the proportion  $p$  of university students who have cheated on an examination. The researcher prepares a box containing 100 cards, 20 of which contain Question A and 80 Question B.

- Question A: Were you born in July or August?
- Question B: Have you ever cheated on an examination?

Each student who is interviewed draws a card at random with replacement from the box and answers the question it contains. Since only the student knows which question he or she is answering, confidentiality is assured and so the researcher hopes that the answers will be truthful. It is known that one-sixth of birthdays fall in July or August.

- What is the probability that a student answers "yes"?
- If  $x$  of  $n$  students answer "yes", estimate  $p$ .
- What proportion of the students who answer "yes" are responding to Question B?

(d) Suppose that 5 people, including you and a friend, line up at random. Let  $X$  be the number of people standing between you and your friend. Write the table of the probability function and the cumulative distribution function for  $X$ .