Discrete Structures

Worksheet 23

Spring 2022
March 29

1. Answer the following questions.

   (a) From a deck of cards, how much more likely is it to choose two spades compared with choosing two queens?

   (b) If there is a 1/3 chance I am wearing blue socks and a 1/5 chance my socks have holes in them, what is the chance that I am wearing blue socks without holes?

   (c) Suppose that in a weekly lottery you have probability .002 of winning a prize with a single ticket. You buy 1 ticket per week for 52 weeks.

      i. What is the probability that you win no prizes?

      ii. What is the probability that you win 3 or more prizes?

2. A florist stocks red roses and white roses. Of these, some have thorns and some do not. Let R be the event that a rose is red, and let T be the event that a rose has thorns. Suppose that the following data has been collected about the roses: 1/4 of the roses have thorns, 1/5 of the red roses do not have thorns, and 3/7 of roses with thorns are white. Consider the experiment of selecting a rose at random. Use the data to find the probability of each of the four events $R \cap T$, $\overline{R} \cap T$, $R \cap \overline{T}$, and $\overline{R} \cap \overline{T}$.

3. Recall the *uniform distribution* on a set (of outcomes of an experiment) of size n assigns probability $1/n$ to each element (outcome).

   (a) Which of the following experiments have the uniform distribution?

   - Numbers rolled when rolling a single die
   - Sums of numbers rolled when rolling two dice
   - Color of sock chosen from a dark drawer of 10 pairs of different colored socks
   - Color of second sock chosen from a dark drawer of 10 pairs of different colored socks

   (b) Suppose that a set S of outcomes of an experiment have the uniform distribution. Find the distribution (probability of each outcome) of:

   - a pair of outcomes, when performing the experiment twice
   - a triple of outcomes, when performing the experiment three times

   Be sure to consider cases when the outcome repeats, and when it doesn't.

4. Complete the following tasks for next lab (Friday). They will be presented at the beginning of the lab.

(a) Three coins are tossed up in the air. What is the probability that...

i. exactly two will land on heads?

ii. at least two will land on heads?

iii. at least one will land on heads or at least one will land on tails?

iv. none will land on tails?

(b) A two-digit number (it cannot have zero as a first digit) is chosen at random. What is the probability that...

i. the number is a multiple of 7?

ii. the number contains two of the same digits?

iii. the number contains two different digits?

iv. the number is even or a multiple of 3?

(c) There is a lottery which has 5 winners for every 1000 tickets sold. What is the probability that...

i. someone buying 15 tickets is a winner?

ii. there is exactly one winner in a group of 10 people that each buy 2 tickets?

iii. there are at least nine winners in a group of 10 people that each buy 2 tickets?

(d) For this question, a "word" is a sequnece of letters (for example, *cart* and *rrtawiqoq* are "words"). A word has length at least 1, and can have length $n \in \mathbf{N}$. Let U be the set of words with no repeated letters. Note that *cart* and *trac*, even though they have the same letters, are different words.

If a word from U is chosen at random (each word has equal probability of being chosen), what is the probability that the word uses all 26?