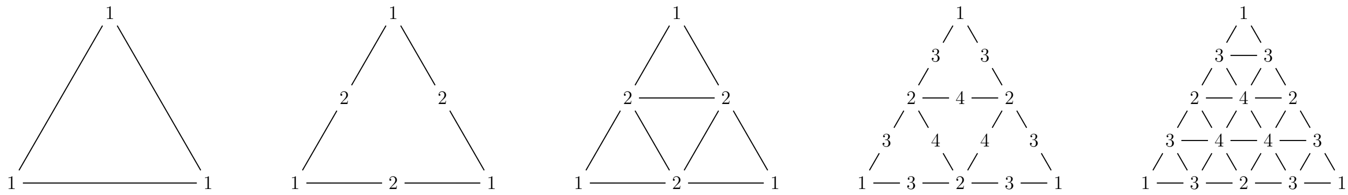
1. Let $f\colon A \to B$ be a function, with $A = \{x_1, \ldots, x_n\}$ and $B = \{y_1, \ldots, y_m\}$.
   (a) How many functions are there from A to B?
   (b) Recall that f is *injective* if $f(p) = f(q)$ implies $a = b$, for all $p, q \in A$. How many injective functions are there from A to B?
   (c) Recall that f is *surjective* if for each $z \in B$, there is some $w \in A$ with $f(w) = z$. What relationship must be true between n and m for there to exist a surjective function from A to B?
   (d) Suppose that $m = n$. How many surjective functions do there exist from A to B?
   (e) Suppose that $m = n - 1$. How many surjective functions do there exist from A to B?

2. Let B_n be the set of all binary strings on the characters `0` and `1` which have length n.
   (a) How many elements are there in B_n?
   (b) How many elements are there in B_n which do not contain the substring `111`?
   (c) How many elements are there in B_n which do not contain the substring `010`?

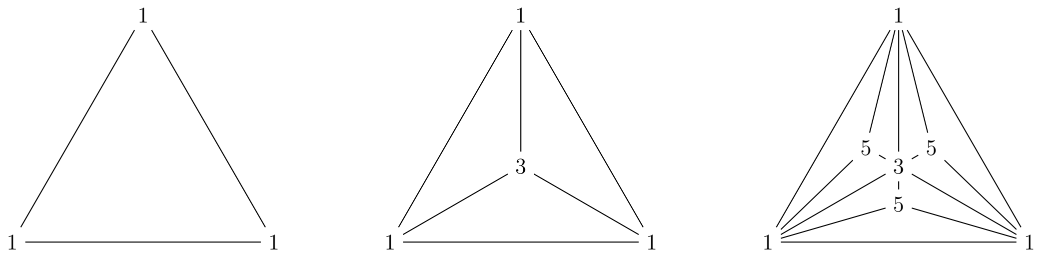
3. This question needs the *pigeonhole principle*. Let $n \in \mathbf{N}$, and consider the set $A = \{n+1, n+2, \ldots, n+100\}$.
   (a) How many subsets $S \subseteq A$ exist, with $|S| = 51$?
   (b) Explain why $n+i$ is relatively prime to $n+i+1$, for every $i = 1, \ldots, 99$.
   (c) Use the pigeonhole principle on the 50 pairs of numbers $(n+1, n+2), (n+3, n+4), \ldots, (n+99, n+100)$ to explain why every $S \subseteq A$, with $|S| = 51$, contains at least one pair of relatively prime numbers.
   (d) Generalize this problem to A having size k. What size should S have then?

4. This question is about the *magic triangle*, also known as the *Pascal*, *Yanghui*, or *Khayyam* triangle, depending on your history.

   (a) Continue as far as possible in the variant of the triangle below. How many new numbers are drawn at each step?



   (b) Continue as far as possible in the variant of the triangle below. How many new edges are drawn at each step?



   (c) What other variants of the triangle can you come up with? What if you use a different shape?

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5. Complete the following tasks for next lab (Tuesday). They will be presented at the beginning of the lab.

   (a) Let $G = (V, E)$ be a graph, with $|V| > 1$. Using the pigeonhole principle, show that there exist two different vertices $u, v \in V$ with $\deg(u) = \deg(v)$.
   Hint: The "boxes" are vertices with the same degree. Begin by showing that two vertices with degree 0 and $|V| - 1$ cannot exist at the same time.

   (b) Let $G = (V, E)$ be a graph with exactly two (no more, no less) vertices u, v of the same odd degree. Show that u must be connected to v. That is, show that $\{u, v\} \in E$.

   (c) This question is about *permutations*.

      i. Into how many unique strings can the string `mississippi` be rearranged?

      ii. Into how many unique orderings can 6 people be placed around a circular table?

   (d) Let P be a polygon with n corners, so that the line between any two corners is inside the polygon. Into how many ways can the polygon be decomposed into triangles?