Discrete Structures

Worksheet 18

Spring 2022
March 8

1. Consider the following forms of license plates around the world:

   - Latvia has the form `AB-0123`
   - The UK has the form `AB01 CDE`
   - Israel has the form `01-234-56`
   - India has the form `AB 01 CD 2345`

   You may assume that only the Arabic numerals (`0,...,9`) and only the English alphabet letters (`A,...,Z`) are allowed.

   (a) How many possible license plates are there for each country?

   (b) Create a table of ratios (rounding to the nearest integer) of your answers from (a).

   (c) If every symbol could be a number **or** a letter, by what factor would each of the countries possibilities increase?

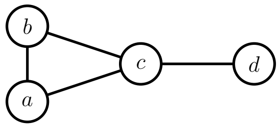
2. How many strings containing the letters `a` and `b` are there:

   (a) of length 12 that contain 7 consecutive letters `a`?

   (b) of length 6 that contain 4 consecutive letters `a` or 3 consecutive letters `b`?

   (c) of length 5 that contain 2 consecutive letters `a` and do not contain 2 consecutive letters `b`?

3. Let $G = (V, E)$ be an undirected graph (so the edges do not have direction). Fix $n \in \mathbf{N}$. How many functions $f: V \to \{1, \dots, n\}$ satisfying $f(u) \neq f(v)$ whenever $\{u, v\} \in E$ are there for $n = 10$ and G as below?



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4. Complete the following tasks for next lab (Friday). They will be presented at the beginning of the lab.

   (a) What percentage of integers between 0 and 10^{10} inclusive are not divisible by any of 6, 14, 11? Make a Venn diagram representing this situation, with circles representing divisibility by each of the given numbers. Indicate the percentage of numbers in each part of the Venn diagram.

   (b) Let $r \in \mathbf{R}_{>0}$, and let T be a triangle with all sides of length r.

      i. Show that two of any five points inside T must be a distance of $r/2$ or less apart.

      ii. Show a counterexample with 4 points inside T and every two of them more than $r/2$ away from each other.

   (c) Using the fact that the number of subsets of $\{1, \dots, n\}$ is 2^n, explain why $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

   (d) Let $A_n = \{1, \dots, n\}$ and let $B = \{0, 1\}$, where $n \in \mathbf{N}$ is fixed.

      i. How many functions are there from A_n to B?

      ii. How many injective functions are there from A_n to B?

      iii. How many surjective functions are there from A_n to B?

   (e) How many different even integers $\geqslant 4000$ and < 7000 have four different digits?