**Discrete Structures**

# Worksheet 17

**Spring 2022**
**March 4**

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1. Answer the following True/False questions:

   (a) Every tree is bipartite.

   (b) There is a tree with degrees $3, 2, 2, 2, 1, 1, 1, 1, 1$.

   (c) There is a tree with degrees $3, 3, 2, 2, 1, 1, 1, 1$.

   (d) If two trees have the same number of vertices and the same degrees, then the two trees are isomorphic.

   (e) If $T$ is a tree with $n$ vertices, the largest degree that any vertex can have is $n - 1$.

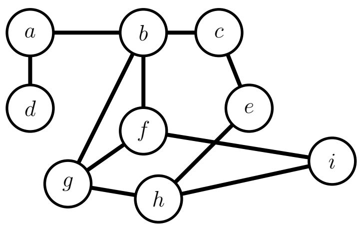
   (f) If $T$ is a binary tree with 15 vertices, then there is a simple path in $T$ of length 6.

   (g) In a binary tree with 16 vertices, there must be a path of length 4.

   (h) If $T$ is a rooted binary tree of height 5, then $T$ has at most 25 leaves.

2. Consider the graph $G = (V, E)$ below.

   $G$ :




   (a) Choose a longest path $P$ of $G$. Call its endpoints $x$ and $y$.

   (b) Choose an edge $e \in P$ which lies in a cycle of $G$. For $G' = (V, E \setminus \{e\})$, find a path connecting $x$ and $y$. Will such a path always exist in $G'$?

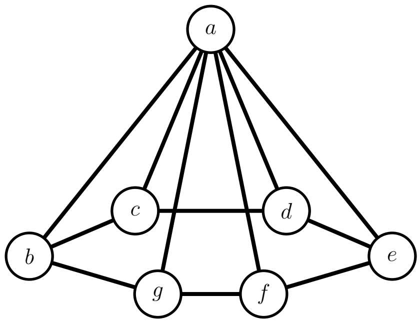
   (c) Choose an edge $f \in P$ which does not lie in a cycle of $G$. For $G'' = (V, E \setminus \{f\})$, try to find a path connecting $x$ and $y$. Will such a path never exist?

3. Let $G = (V, E)$ be a graph with average degree equal to 3.

   (a) Show that $G$ must have at least 4 cycles.

   (b) Is the bound from part (a) above the highest upper bound? That is, must every such $G$ contain at least 5 cycles? At least 6 cycles?

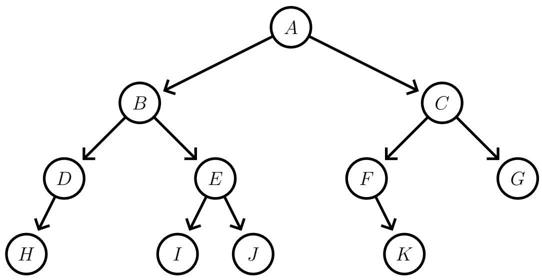
4. Consider the graph $G = (V, E)$ below.




For every $n \in \mathbf{N}$, how many paths of length $n$ in $G$ are there that go through the top vertex $a$ at most once?

5. Consider the binary tree $T$ below.

$T:$




(a) List the vertiex labels, if they are visited in preorder, inorder and postorder sequence.

(b) Draw a tree that is the mirror image of the tree $T$. List the vertices of this mirrored tree in preorder, inorder and postorder sequence.

(c) How are the sequences of parts (a) and (b) above related? Are the sequences reverse images of ech other?

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**5.** Complete the following tasks for next lab (Tuesday). They will be presented at the beginning of the lab.

(a) Let $G = (V, E)$ be a graph with $V = \{v_1, \ldots, v_n\}$ and average degree $\frac{\deg(v_1)+\cdots+\deg(v_n)}{n} > 2$.

i. Show that $G$ must contain at least 2 cycles.

ii. Show with an example that it is possible for $G$ to contain exactly 2 cycles.

(b) i. Find a graph $G$ which has an Eulerian circuit, but no Hamiltonian circuit.

ii. Find a graph $H$ with has a Hamiltonian circuit, but no Eulerian circuit.

(c) Prove the following statement or show that it is false:

*Given a sequence of positive integers $d_1, \ldots, d_n$ that sum to $2(n-1)$, there exists a tree with $n$ vertices $v_1, \ldots, v_n$ with $\deg(v_i) = d_i$, for all $i$.*