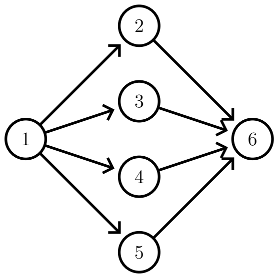
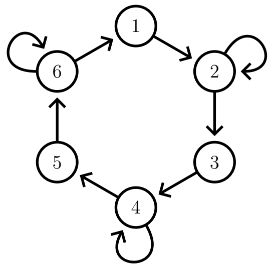
Worksheet 16

1. Consider the directed graphs G and H:

$G =$ 

$H =$ 

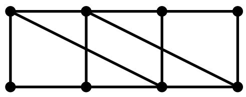
   (a) Construct the adjacency matrices M_G and M_H for the graphs.

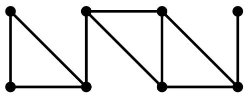
   (b) Compute the matrices $M_G \cdot M_H$, $M_H \cdot M_G$ and $M_H^T \cdot M_G$. You may use a computer.

   (c) Construct the directed graphs from the matrix products of part (b). What is the relationship between it and G, H?

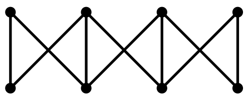
2. This question is about *isomorphisms*.

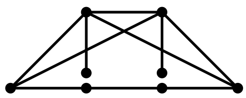
   (a) Using an argument about edges, explain why the following graphs are not isomoprhic.



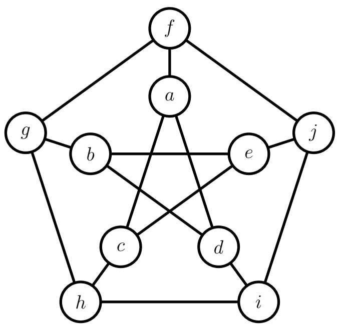


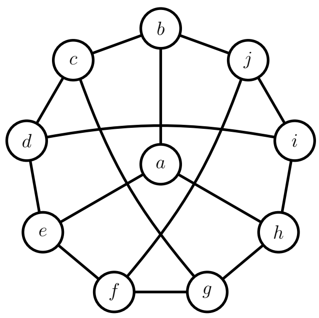
   (b) Using an argument about degrees, explain why the following graphs are not isomoprhic.





   (c) Describe explicitly an isomorphism between the following two graphs.

$G_1 =$ 

$G_2 =$ 

3. This question is about *bipartite* graphs. Let $G = (V, E)$ be a bipartite graph.

   (a) Compute the density of G if $G = K_{n,n/2}$ for $n \in \mathbf{N}$ even.

   (b) Show by construction that Q_n is bipartite.

   (c) Prove that $|E| \leqslant \frac{|V|^2}{4}$.

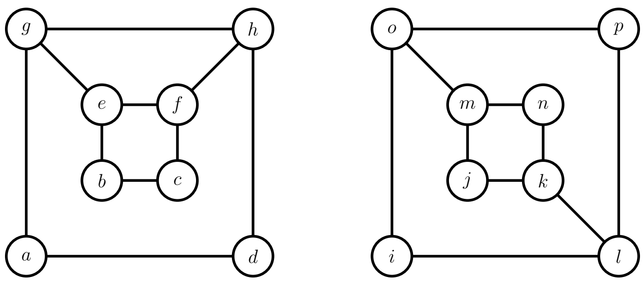
4. This question is about the *handshaking theorem*. Let $G = (V, E)$ be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

---

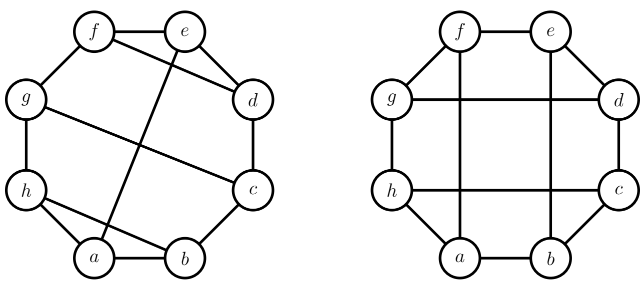
5. Complete the following tasks for next lab (Friday). They will be presented at the beginning of the lab.

   (a) For each pair of graphs below, use paths or cycles to show they are not isomorphic, or describe an explicit isomorphism between the two.

i.



ii.



   (b) Let $G = (V, E)$ be a graph with $v \in V$ having odd degree. Explain why there is a path in G from v to another vertex of odd degree.
   Hint: Use the Handshake theorem.

   (c) Let $G = (V, E)$ be a graph with $|V| = 5$ and $|E| = 7$. Show that for any two vertices in V, there is always a path of length 1 or 2 between them.