Discrete Structures

Worksheet 1

Spring 2022
January 4

Recall the following logical symbols:

$\wedge$	and	$\exists$	there exists	$\rightarrow$	implies
$\vee$	or	$\forall$	for all, for each	$\leftrightarrow$	if and only if
$\neg$	not	$\in$	belongs to	( ) ,	*gives order to parts of proposition*

This worksheet will use the sets **N** (the natural numbers), **Z** (the integers), and **R** (the real numbers.

---

1. **Warm up:** Let P and Q be propositions and $x \in X$ some number. Consider the sentences on the left and the sequences of logical symbols on the right.

If P is true, then $(P \vee Q)$ is true	$Q \vee (\exists\, x \in X,\ \neg P)$
If P is true, then $(P \wedge Q)$ is true	$P \leftrightarrow (\neg P)$
P is true for every $x \in X$	$P \rightarrow (P \vee Q)$
Either Q is true or P is false for some $x \in X$	$\forall\, x \in X,\ P$
P implies Q is the same as $\neg Q$ implies $\neg P$	$P \rightarrow (P \wedge Q)$
P is the same as $\neg P$	$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

   (a) Connect the sentences on the left with their matching symbols on the right.

   (b) For each sentences, come up with another sentence in English that has the same meaning.

   (c) What is the difference between the last two sentences on the left?

2. Consider the five propositions below.

   (P_1) I like pineapple on my pizza.

   (P_2) All odd-numbered propositions are false.

   (P_3) All even-numbered propositions are true.

   (P_4) At least one of P_2 or P_3 is true.

   (P_5) If P_1 is false then P_2 is true.

   Assign the values *True* and *False* to each of the above five propositions so that there are no contradictions, or say why it is not possible.

   Hint: Some of these propositions refer to other propositions on the list. Notice that if P_3 is true then all even-numbered propositions must be true, and so P_2 must be true. The truth of P_2 implies all odd-numbered propositions are false, and so P_3 is false. So if P_3 is true then it must also be false. This contradiction means P_3 must not be true.

3. There are 100 propositions written on a piece of paper, as below.

    (1) Exactly 1 of the propositions on this paper is false.

    (2) Exactly 2 of the propositions on this paper are false.

    (3) Exactly 3 of the propositions on this paper are false.

    ...

    (100) Exactly 100 of the propositions on this paper are false.

    Which of these propositions are true, and which ones are false?

4. Rewrite the following propositions using only symbols.

    (a) Every natural number is a rational number.

    (b) Not every real number is a natural number.

    (c) Dividing a real number by a rational number always gives a positive or negative natural number.

5. Complete the following truth table.

P	Q	$(P \to Q) \leftrightarrow (\neg Q \to \neg P)$	$(P \vee Q) \vee (\neg P \vee Q)$	$(P \wedge \neg P) \vee (Q \wedge \neg Q)$
T	T			
T	F			
F	T			
F	F			

6. Prove the following statements by construction:

    (a) For every function $f\colon \mathbf{R} \to \mathbf{R}$, theere exists a real number c_f such that the function $g\colon \mathbf{R} \to \mathbf{R}$ defined by $g(x) = f(x) + c_f$ satisfies $g(0) = 0$.

    (b) There is a function $f\colon \mathbf{R} \to \mathbf{R}$ that is not continuous at any $x \in \mathbf{R}$.