1. **Warm up:** Answer the following True / False questions.

   (a) If $|A| = n$ for a set A, then the number of subsets of A is 2^n.

   (b) If $|A| = 2^n$ for a set A, then the number of subsets of A is 4^n.

   (c) The number of ways to choose a pair of cards from a deck is $104 = 2 \cdot 52$

   (d) If $f \colon \mathbf{N} \to \mathbf{N}$ is surjective, then $|S| = |\{f(s) \ : \ s \in S\}|$ for any $S \subseteq \mathbf{N}$.

2. What percentage of integers between 0 and 10^{10} inclusive are not divisible by any of 6, 14, 11? Make a Venn diagram representing this situation, with circles representing divisibility by each of the given numbers.

3. Let $A_n = \{1, \ldots, n\}$ and let $B = \{0, 1\}$, where $n \in \mathbf{N}$ is fixed.

   (a) How many functions are there from A_n to B?

   (b) How many injective functions are there from A_n to B?

   (c) How many surjective functions are there from A_n to B?

4. Let $G = (V, E)$ be an undirected graph (so the edges do not have direction). Fix $n \in \mathbf{N}$. How many functions $f \colon V \to \{1, \ldots, n\}$ satisfying $f(u) \neq f(v)$ whenever $\{u, v\} \in E$ are there for $n = 10$ and G as below?
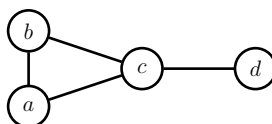
5. How many strings containing the letters `a` and `b` are there:

   (a) of length 12 that contain 7 consecutive letters `a`?

   (b) of length 6 that contain 4 consecutive letters `a` or 3 consecutive letters `b`?

   (c) of length 5 that contain 2 consecutive letters `a` and do not contain 2 consecutive letters `b`?

6. Let $r \in \mathbf{R}_{>0}$, and let T be a triangle with all sides of length r.

   (a) Show that two of any five points inside T must be a distance of $r/2$ or less apart.

   (b) Show a counterexample with 4 points inside T and every two of them more than $r/2$ away from each other.

7. Consider the following forms of license plates around the world:

- Latvia has the form AB-0123
- The UK has the form AB01 CDE
- Israel has the form 01-234-56
- India has the form AB 01 CD 2345

You may assume that only the Arabic numerals (0,...,9) and only the English alphabet letters (A,...,Z) are allowed.

(a) How many possible license plates are there for each country?

(b) Create a table of ratios (rounding to the nearest integer) of your answers from (a).

(c) If every symbol could be a number **or** a letter, by what factor would each of the countries possibilities increase?

8. Computers count time in 31 binary digits, with time 0 being Jaunary 1, 1970, 00:00. For the purposes of this question, you may assume that all years have 365 days.

(a) Write a function in Python that converts binary time into regular time. That is, it takes in a string representing binary time and outputs time in the 24-hour manner. You may use the `int` function as below.

```
def regular_time(bin_time):
    dec_time = int(bin_time,base=2)
    seconds = ...
    minutes = ...
    return str(minutes).zfill(2) + ":" + str(seconds).zfill(2)
```

For example, `regular_time("101")` should return the string `"00:05"`.

(b) Write a function in Python that converts binary time into the year and day of the year. Below is an example of how such a function would look like.

```
def year_day(bin_time):
    dec_time = int(bin_time,base=2)
    day = ...
    year = ...
    return "year " + str(year) + " day " + str(day)
```

For example, `year_day("1011110000111000110011110000")` should return the string `"year 1976 day 95"`.

(c) Using the functions from parts (a) and (b):
  i. When did it become necessary to use 31 digits instead of 30?
  ii. When will it be start to be impossible to use 31 digits?

(d) How far off (in terms of days, hours, minutes) is your computed time compared with the actual time? The following code will get you the current time in binary:

```
import time
f"{int(time.time()):b}"
```