Data Structures
BITL3

Worksheet 11

Fall 2022

24 November 2022

This worksheet uses the following definitions.

- **height of a node:** length of the longest path from a node to a leaf below it
- **height-balanced (AVL) tree**: satisfies $|\text{height(right child)} - \text{height(left child)}| \leqslant 1$

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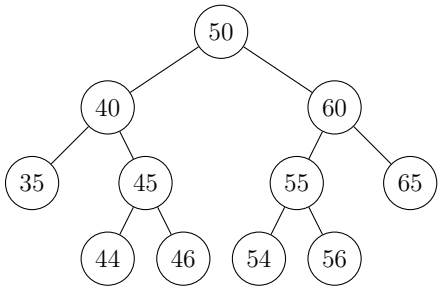
1. In the **AVL tree** below left, perform the operations below right, in the given order. Make sure to rebalance (if necessary) after every operation.

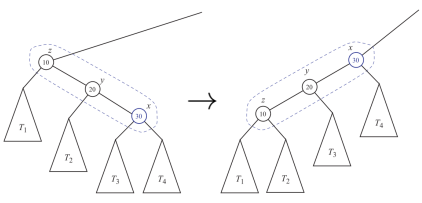


(a) insert 43
(b) insert 66
(c) insert 67
(d) remove 60

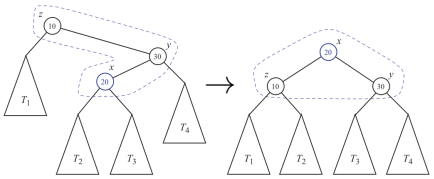
Recall that rebalancing is done in terms of **rotations**. The x**-over-**y rotation of T, for nodes x, y of T where x is a child of y, is a new tree T' identical to T, except for:

- if $T.y.parent = z$, then $T'.x.parent = z$ and $T'.y.parent = x$
- if $T.y.leftchild = x$, then $T'.y.leftchild = T.x.rightchild$
- if $T.y.rightchild = x$, then $T'.y.rightchild = T.x.leftchild$

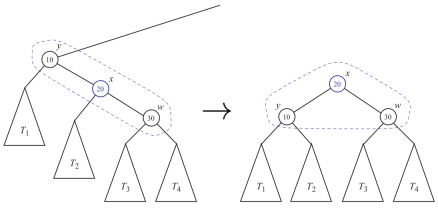
2. This question is about **splaying**. If x is a node in a tree T with parent y and grandparent z, to *splay* x means to make x the root node of T by sequences of the following actions (and their symmetrical analogues).



zig-zig



zig-zag



zig

(a) How many steps does it take to splay 23 in the tree T on the right? Draw T at each step.

(b) Remove the key 22 from the tree on the right. What are the two possible trees that result, using the two approaches (largest of left subtree, smallest of right subtree)?