Data Structures

Worksheet 6

BITL2

21 October 2021

Recall that a **red-black** tree satisfies the following properties:

- The root and all leaves are black
- Both children of a red node are black
- All leaves have the same black depth (black ancestors)

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1. **Warm up 1:** Answer the following True / False statements about red-black trees.

   (a) A subtree of a red-black tree is itself a red-black tree.

   (b) The sibling of a leaf is either a leaf or it is red.

   (c) Every red-black tree is an AVL (height-balanced) tree.

2. A **matching** of a tree is a subset of the edges of a tree so that no two edges share a vertex. A matching is **perfect** if every vertex of the tree is incident to exactly one edge of the matching.

   (a) Does a complete binary tree always have a matching? Which do and which do not?

   (b) For a binary tree of height h, what is the largest number of nodes it can have to have a perfect matching?

   (c) For an n-ary tree of height h, what is the largest number of nodes it can have to have a perfect matching?

3. Recall the **insertion** sort, **selection** sort, **merge** sort, and **quick** sort algorithms.

   (a) Which of these are in-place algorithms? Deterministic algorithms?

   (b) If an input list is already sorted, which algorithm will be fastest?

   (c) How many comparisons do each of insertion and selection sort do in each step, when sorting the list below? A comparison is when two list elements are compared in size.

1	4	19	5	15	10	2	3

step	1	2	3	4	5	6	7	8
comparisons for insertion sort								
comparisons for selection sort								

   (d) Draw all the steps that merge sort would take on this list.

   (e) For quick sort, which of the values in the list are good pivots? Draw all the steps that quick sort would take on the list, if the pivot is element number $\lfloor \frac{\text{length of list}}{2} \rfloor$.