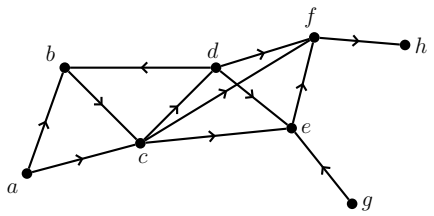
Data Structures
BITL1

Worksheet 12

Fall 2020

3 December 2020

1. **Warm up:** Answer the following questions about the directed graph G below.



   (a) Which vertices span the largest strongly connected component?

   (b) An *n-clique* is an ordered set of vertices $v_1, \ldots, v_n$ such that $i < j$ implies (v_i, v_j) is an edge. Which vertices span the largest directed n-clique?

   (c) If Dijkstra's algorithm is initiated at a, how many vertices will have weight 3 when the algorithm finishes? You may assume all edges have weight 1.

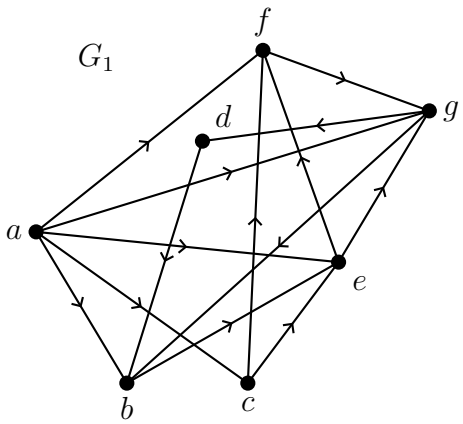
2. Without looking at your notes, give definitions of the following terms, in your own words.

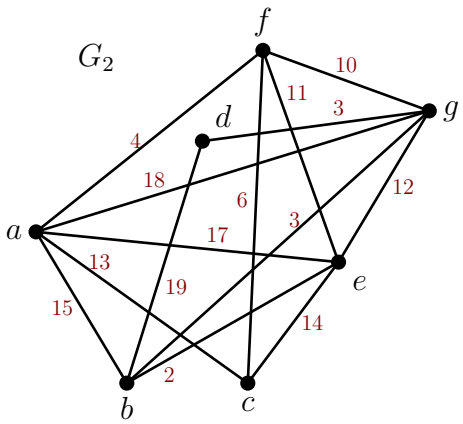
   (a) partition of a graph

   (b) weighted directed graph

   (c) subtree of a graph

3. Consider the following graphs.





   (a) Compute the output of the topological sorting algorithm on G_1, with vertices initially ordered by their alphabetical label.

   (b) Compute the output of Kruskal's algorithm on G_2, but stop it before edges of weight $\geqslant 10$ are added.

4. Recall the **travelling salesperson problem**, which tries to find a spanning cycle of smallest weight in an undirected weighted graph $G = (V, E)$. For each of the conditions below, what can you say about a / any solution to the travelling salesperson proplem?

   (a) Suppose you know that G has a cut edge. A *cut edge* of $G = (V, E)$ is an edge $e \in E$ such that $G' = (V, E \setminus \{e\})$ is disconnected.

   (b) **Bonus:** Suppose you know $\deg(v) + \deg(u) \geqslant |V|$ whenever $\{u, v\} \notin E$, and $|V| \geqslant 3$.