Data Structures
BITL1

Worksheet 11

Fall 2020

26 November 2020

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1. **Warm up:** Answer the following questions.

   (a) What does a connected graph for which BFS and DFS reaches a given node node in the same number of steps look like?

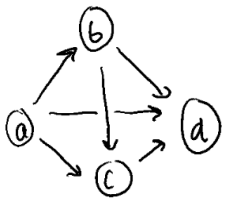
   (b) How many new edges are added to get the transitive closure of the directed graph below? It has 100 vertices.

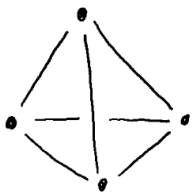
   $$\bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \quad \cdots \quad \longrightarrow \bullet \longrightarrow \bullet$$

   (c) True or False: A directed graph is strongly connected if and only if its transitive closure is strongly connected.

2. Recall a **weighted graph** is a graph $G = (V, E)$ with a function $w \colon E \to \mathbf{R}$, called the *weight* of the edge e.

   (a) Assign different weights to each edge of the directed graph on the left, so that for any two paths p_1, p_2 from a to d, $|weight(p_1) - weight(p_2)| \leqslant 1$. Is it possible to do this with just integer weights?

   (b) Suppose each edge of the undirected graph on the right is randomly assigned a direction. What is the probability that the directed graph has a cycle?

3. A graph is k**-regular** if every vertex has degree k.

   (a) Let G be a connected 2-regular graph with n vertices. What is the length of a minimal spanning tree of G?

   (b) Let G be a connected 3-regular graph. If G has 4 vertices, how many cycles does it have?

   (c) Can you construct a connected 3-regular graph with 8 vertices?

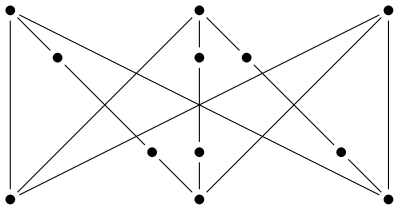
4. Draw the directed graphs associated to ther matrices A, B below, then to the graph associated to $A \cdot B$. Multiply the matrices using modulo 2 multiplication.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. The "Knight's move graph" $KM(4)$ has 16 vertices, denoted (u, v) for $u, v \in \{1, 2, 3, 4\}$, and edges $(a, b)-(x, y)$ if and only if $(a - x)^2 + (b - y)^2 = 5$.

   (a) Draw this graph.

   (b) **Bonus:** Find the following graph as a subgraph of $KM(4)$.



   This graph is (a subdivision of) the graph $K_{3,3}$. Having this graph as a subgraph means $KM(4)$ cannot be drawn without edges crossing each other.

6. **Bonus:** A **triangle-free** graph is a graph without any cycles of length 3. What is the largest number of edges for a triangle-free graph with 8 vertices?