Data Structures
BITL1

Worksheet 10

Fall 2020

19 November 2020

---

1. **Warm up:** Describe the following terms in your own words.
   (a) in-place sorting algorithm
   (b) comparison-based sorting algorithm
   (c) amortized analysis
   (d) deterministic algorithm

2. Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, and $X = \{1, 2, 4, 8, 12, 16, 20, 22\}$ be sets.
   (a) Using only union $\cup$, intersect $\cap$, and subtract $-$ with A and B, make a set of:

   i. size 1   ii. size 2   iii. size 4   iv. size 0

   (b) How many different partitions of X into sets of size 3,3,2 are there?
   (c) Considering only size of a partition, how many different partitions are there for X, if each partition must have size 2 or 3?

3. A **graph** $G = (V, E)$ is a set V and another set E, where every element of E is a 2-element set $\{v_1, v_2\}$, for $v_1, v_2 \in V$. Consider the following algorithm on a graph $G = (V, E)$, that takes as input two elements $\texttt{head}, \texttt{tail} \in V$ and a non-negative integer ℓ.

```
1 define f(head, tail, ℓ) :
2   E′ ← {e ∈ E : head ∈ e, tail ∉ e}
3   if E′ = ∅ :
4     return ℓ
5   else:
6     return max_{{head,next}∈E′} {f(next, head, ℓ + 1)}
```

   (a) If $f(v, w, 0)$ is called for any $\{v, w\} \in E$, what result will this algorithm produce?
   (b) For what G and $e = \{v, w\}$ will $f(v, w, 0)$ terminate, and for what G, e will it not?
   (c) A **directed graph** is a graph $G = (V, E)$ where every element of E is an ordered tuple (v_1, v_2). Suppose line 2 in the above algorithm is changed to the following:

   $$2 \qquad E' \leftarrow \{e \in E \;:\; e = (\texttt{head}, v), v \in V\}$$

   For what directed G and v will $f(v, v, 0)$ terminate, and for what G, v will it not?
   (d) A **weighted graph** is a graph $G = (V, E)$ with a function $w\colon E \to \mathbf{R}$, called the *weight* of the edge e. Suppose line 6 in the above algorithm is changed to the following:

   $$6 \qquad \textbf{return} \max_{\{\texttt{head},\texttt{next}\}\in E'} \{f(\texttt{next}, \texttt{head}, \ell + w(\{\texttt{head}, \texttt{next}\}))\}$$

   Do there exist graphs for which the original algorithm did not terminate, but with this change to line 6 will not keep increasing the maximum of the set in this line?