Data Structures
BITL1

Worksheet 7

Fall 2020

29 October 2020

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1. **Warm up:** Answer the following questions.
   (a) True / False: A heap has keys in a perfect binary tree.
   (b) What is an advantage of using heaps for sorting?
   (c) Describe `push` and `pop` methods of a FIFO queue in terms of a priority queue.

2. Recall the **binary search tree** data structure. Suppose a map data structure has keys stored in a binary search tree.
   (a) What is the difference between a binary search tree and a regular binary tree?
   (b) What is the running time of the `find` method, where n is the number of keys?
   (c) Suppose you are given the following binary search tree: $\{9, 6, 12, 2, 8, 10, 18, 1, 7\}$. How many steps would it take to turn it into a heap? Draw the tree at each step.
   (d) If you have a binary search tree of length n, what is the time complexity of turning it into a heap?

3. This question is about **hashing**.
   (a) Suppose you have three string for keys `big`, `dig`, `rig`. Using only the function `letterindex(`ℓ`)` which returns the index of a letter in the English alphabet, as well as any algebraic functions, create hash functions which:
      i. Send the three keys to different integer values.
      ii. Send the three keys to the same integer value (do not use a constant function).
      iii. Send every three-letter string to a different integer value.
   (b) Let $m \in \mathbf{N}$ be the size of a hash table. Given a hash function $h(k) = 57k \mod m$, what is the probability that two different integers in the set $\{1, 2, \ldots, 10\}$ map to the same hash? Use $m = 10, 100, 1000$ to motivate your answer.
   (c) Consider a hash function $h(k) = \lfloor 1000 \cdot (kA \mod 1) \rfloor$ for $A = (\sqrt{5} - 1)/2$. Compute the locations to which the keys $61, 62, 63, 64$, and 65 are mapped.