

15 October 2020

1. **Warm up:** Answer the following questions about tree structures.
 - (a) What is the shortest and longest path between two level ℓ nodes in a binary tree?
 - (b) What is the smallest and largest numbers of leaves a height h binary tree can have?
2. Let T be a tree.
 - (a) If T is binary and full with height h , how many nodes does it have?
 - (b) If T is n -ary and full with height h , how many nodes does it have?
 - (c) Answer the warm up questions, but for an n -ary tree instead of a binary tree.
3. A **matching** of a tree is a subset of the edges of a tree so that no two edges share a vertex. A matching is **perfect** if every vertex of the tree is incident to exactly one edge of the matching.
 - (a) Does a complete binary tree always have a matching? Which do and which do not?
 - (b) For a binary tree of height h , what is the largest number of nodes it can have to have a perfect matching?
 - (c) For an n -ary tree of height h , what is the largest number of nodes it can have to have a perfect matching?
4. Suppose a binary tree has been implemented using an array.
 - (a) Given a node of the tree at level ℓ and position p from the left, what is the index of the array at which this node exists? Both ℓ and p start at 0.
 - (b) Describe the binary tree ADT methods using the stack ADT methods.