Homotopy theory for topological data anaylsis

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Abstract: I will describe some common starting points to building the theoretical foundations of topological data analysis through homotopy theory.

0.1 Facts

Year / advisors + area / job post-phd / advice

0.2 Motivation

Let X be a space. Path in space.

Q1: How many different paths (with common endpoints) in X are there?
"different": definition of homotopic paths

Q1.1: How many different loops in X are there?
definition of fundamental group (Groves, Shipley)

Obs: $[0,1]$ can be represented as the convex hullk of $\{(1,0),(0,1)\} \subseteq \mathbf{R}^2$.
Let Δ^n_{top} = convex hull of $n+1$ unit vectors in $\mathbf{R}^{n+1}$
Q2: How many different continuous functions $\Delta^n_{top} \to X$ are there?
"different": note that $\Delta^n_{top} \times [0,1] \cong \Delta^{n+1}_{top}$
definition of singular category (Antieau, Shipley)

Suppose: X has a nice shape, has decomposition into pieces of fixed dimension, like Δ^n_{top} with faces
stratification: continuous map $X \to (A, \leqslant)$
Q2.1: How many different continuous functions $\Delta^n_{top} \to X$ that respect the stratification of X are there?
subcategory $\text{Sing}_A(X)$

0.3 Research specifics

M is a Riemannian manifold
$\text{Ran}(M)$ is the space of finite subsets of M, topology induced by Hausdorff distance: (Dumas, Whyte)

$$d_H(P,Q) = \min\left\{\epsilon : Q \subseteq \bigcup_{p\in P} B(p,\epsilon),\ P \subseteq \bigcup_{q\in Q} B(q,r)\right\}$$

View $P \in \text{Ran}(M)$ as the vertices of a simplicial complex. Build it by some paramater $r \in \mathbf{R}_{\geqslant 0}$
Obs: When P or r move slightly, same isomorphism class of simplicial complex
Use locally constant sheaves to describe this phenomenon.
To every (nice) open set associate:

Visualization: Contractible path in $\text{Ran}(M)$
Contractibility theorems