This talk goes through pages 38-48 in Casson and Bleiler's "Automorphisms of surfaces"

F is a closed hyperbolic surface (recall $\mathbf{H}^2$ may be tiled with F)

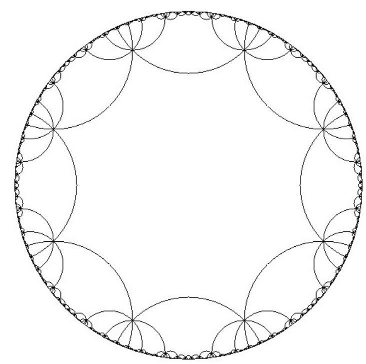
0.1 Part 1: Setup

Definitions: A *complete geodesic* on $\mathbf{H}^2$ is a diameter or a circle intersecting at 90°. A *geodesic* on F is the image of a complete geodesic on $\mathbf{H}^2$ via the tiling. A *lamination* L of F is a non-empty closed subset of F that is a disjoint union of geodesics, called *leaves* of the lamination.

Example:

- draw fundamental domain of genus 2 surface
- label edges $aba^{-1}b^{-1}cdc^{-1}d^{-1}$
- since one vertex, angle is $2\pi/8$ at corners
- draw genus 2 surface in $\mathbf{R}^3$

- draw geodesic from a to a^{-1} centers (closed)
- draw geodesic from c to c^{-1} centers (closed)
- draw geodesic approximating both (not closed)



L is a geodesic lamination of F

Geodesics are unoriented.

Lemma 3.1: Geodesics are (at least) C^1

Lemma 3.2: L is non-empty, disjoint union $\implies \overline{L}$ is a lamination
Ensures that limit of non-closed geodesics is still geodesic.

Lemma 3.3: a) L is nowhere dense in F, b) L may be expressed uniquely

0.2 Part 2: Structure / lifting

Definitions: For any space X, the *Hausdorff distance* on $\mathbf{P}(X)$ is defined as

$$d(U,V) \leqslant \epsilon \quad \Longleftrightarrow \quad U \subset \bigcup_{v \in V} B(v,\epsilon),\ V \subset \bigcup_{u \in U} B(u,\epsilon).$$

Set the *projective tangent bundle* of F to be $PT(F) \xrightarrow{p} F$ with $p^{-1}(x) = \{(x,\sigma)\ :\ \sigma$ is a geodesic on F with $|\sigma| = 2, \sigma(1) = x\}$. Note that $p^{-1}(U) \cong U \times \mathbf{RP}^1$.

$\mathbf{P}^c(X) = \{Y \in \mathbf{P}(X) \setminus \emptyset\ :\ Y \text{ is closed}\}$
$\Lambda(F) = \{\text{geodesic laminations on } F\} \subset \mathbf{P}^c(F)$

Theorem 3.4: $(\Lambda(F), d)$ is (sequentially) compact

Now lift everything we have learned so far:

$$\widehat{\gamma} \dashrightarrow \{(x,\sigma) \in PT(F)\ :\ \sigma \subset \gamma\}, \qquad \widehat{L} \dashrightarrow \{\widehat{\gamma}\ :\ \gamma \in L\}, \qquad \widehat{\Lambda}(F) \dashrightarrow \{\widehat{L}\ :\ L \in \Lambda(L)\}$$

$$\begin{array}{lccccccccl}
p(x,\sigma) = x & \widehat{\gamma} & \in & \widehat{L} & \subset & \widehat{\Lambda}(F) & \subset & \mathbf{P}^c(PT(F)) & & p_*(\widehat{L}) = L \\
p(\widetilde{\gamma}) = \gamma & \downarrow p & & \downarrow p & & \downarrow p_* & & \downarrow p_* & & p_*(\{(x,\sigma)\}) = \{x\} \\
 & \gamma & \in & L & \subset & \Lambda(F) & \subset & \mathbf{P}^c(F) & &
\end{array}$$

The map p is 1-1 and onto. This implies that p_* is a homeomorphism.

Lemma 3.5: $(\Lambda(F), \mathbf{P}^c(F)) = (\Lambda(F), \mathbf{P}^c(PT(F)))$ as topological spaces with base of topology

Definitions: A leaf γ of L is *isolated* if for every $x \in \gamma$ there exists $U \subset F$ such that $U \setminus \gamma$ has two components. A lamination L of F is *isolated* if there exists $\epsilon > 0$ such that $d(L,M) \geqslant \epsilon$ for all $M \neq L$ in $\Lambda(F)$. Set $L' = L \setminus \{\text{isolated leaves}\}$.

Example: Laminations with leaves that are not closed on F are not isolated (taking M to have leaves that are the limits of non-closed leaves in L).

Lemma 3.6: $L' = \emptyset \implies$ a) L is isolated in $\Lambda(F)$, b) $L = \bigcup_{i=1}^{n} \gamma_i$ for every γ_i simple closed