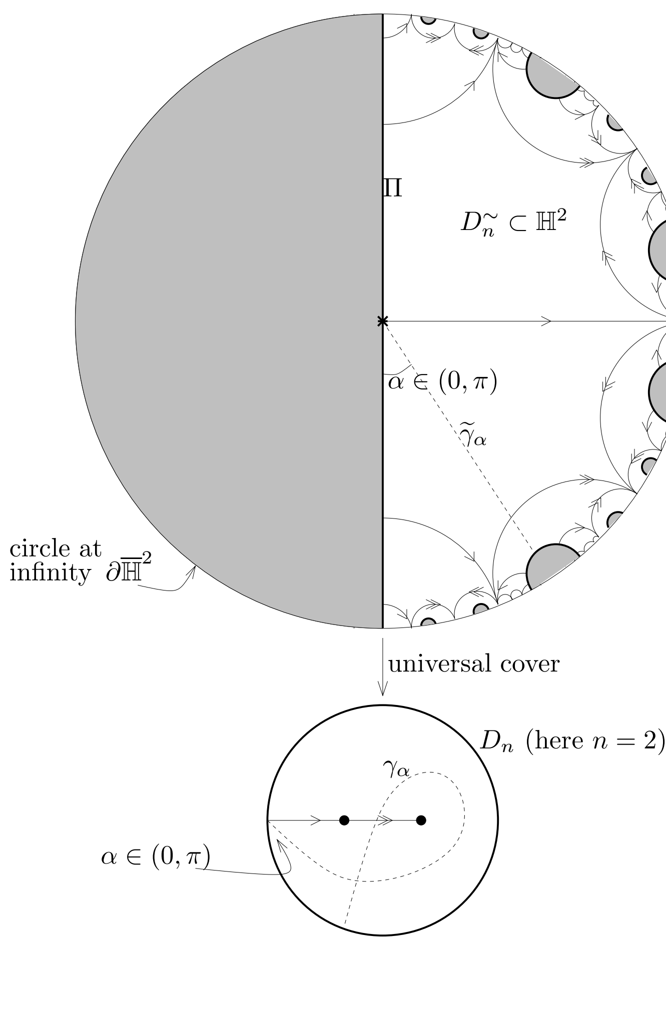
Tiling the hyperbolic plane

Aside: Based off Jake's talk. Posed more questions than answered.

1. Recap + motivation

- Ordering braids. Start with n-punctured disk D_n.
- Fix basepoint of D_n on ∂D_n. Choose path (geodesic) on D_n from basepoint (best to separate pts).
- Apply braid moves. Compare which higher/lower. This is left-ordering ($g < h$ implies $fg < fh$).
- For example:

- Recall huge picture of $\mathbf{H}^2$ Jake drew. This was part of proof that this gives ordering.
- Pic was lift of D_n with trivial curve diagram and geodesic to universal cover of D^2
- Universal cover embeds in $\mathbf{H}^2$.



- Question **Q1**: How much of $\mathbf{H}^2$ is not covered by the universal cover?
    - Look at boundary S^1_∞ of $\mathbf{H}^2$.
    - Punctures at ∞ are Cantor set (maybe fat?). Intervals at ∞ are not in universal cover.
        - Area in UC is infinite (infinite finite area shapes).
        - Area not in UC is infinite (integral along intervals at ∞ will be ∞).
- Poorly posed question.
- Better question **Q2**: What is the ratio of (area in UC)/(area not in UC)?
    - Guess: 0. Problem: How to express area in UC?

2. Calculating area on $\mathbf{H}^2$

- **Q3**: Is there a canonical way to draw universal cover?
    - *Draw first disk. Length on edge? Where to put edges?* Do these things matter? Shouldn't.
    - Guess: All ways are fine. May have to adjust metric (*which way works for standard metric?)*
- *How to express area not in UC?*
    - *Label as X. Should be $\int_X \omega$, for ω an appropriate metric.*
- *Using Poincare disk model, so* $\omega = 4\dfrac{dx^2 + dy^2}{(1 - \|(x, y)\|^2)^2}$, *giving* $\mathbf{H}^2$ *coordinates from unit sphere in* $\mathbb{R}^2$
    - *Problem of infinite area justified here. Radius=1*
    - *Problem: parametrize area not in UC (X)*